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SKEW PLANE SHOCK WAVE AT THE INTERFACE BETWEEN TWO POLYTROPIC GASES. A HEAVY–LIGHT GAS SYSTEM

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In [Inzh.–Fiz. Zh., **72**, No. 2, 208–215, 216–225 (1998)] *the problem of break decay that occurs when a plane skew shock wave (SSW) arrives at the interface between two polytropic gases of different density (the polytrope indices of both gases remain constant in the entire region of interaction) from the side of the less dense of them is considered in detail. The change in the stationary shock-wave configuration with an increase in the angle of incidence from 0 to π/2 is tracked. In the present work, consideration is given to the development of a stationary shock-wave configuration in the case where the SSW emerges at the interface from the side of the more dense gas. Problems associated with boundary layers and mixing of contacting gases were not considered.*

In [1, 2] it was noted that in the emergence of an SSW at the interface between two polytropic gases from the side of the less dense of them, for small angles of incidence ($\varphi < \varphi_t$ and $\varphi < \varphi_c$), we should observe a regular interaction where a reflected shock wave occurs at the contact point (CP) between the SSW and a refracted shock wave (RrSW) and propagates over the "upper" (lighter) gas. The fact that here the conditions of interaction correspond to the occurrence of a reflected shock wave (RlSW) rather than a reflected rarefaction wave (RlRW) can be ascertained by tracking the dynamics of the change in the pressure and the component of the gas velocity that is normal to the interface following the refracted shock wave and the SSW using the procedure adopted in [1] for calculating these parameters. Clearly the occurrence of the reflected shock wave must correspond to the following relations for these quantities: $p_{1,\mathrm{H}} < p_{1,\mathrm{L}}$ and $v_{1,\mathrm{H}} > v_{1,\mathrm{L}}$, while the appearance of the reflected rarefaction wave must be preceded by $p_{1,\mathrm{H}} > p_{1,\mathrm{L}}$ and $v_{1,\mathrm{H}} < v_{1,\mathrm{L}}$. Plotting on one graph all possible values of these parameters behind the SSW front and the changes in these quantities that correspond to a given fixed $\varphi_{1,\mathrm{H}}$ behind the front of the refracted shock wave for any value of the angle $\varphi_{1,\mathrm{L}}$ (Fig. 1), we note that for a light→heavy gas system, there are exclusively regions of the relation of the parameters that allow the occurrence of a reflected shock wave (the arrow is used as an indicator that the shock wave first moves over the gas preceding this symbol). Figure 2 gives such diagrams for an air–krypton system for $\varphi_{1,\mathrm{H}}$ = 30, 45, and 60°, respectively. This picture is characteristic of any light→heavy gas system. In considering the opposite combination (heavy→light gas) (Fig. 3), we observe only the presence of regions that allow the occurrence of a reflected rarefaction wave (also independently of the specific choice of the gases).

Thus, the heavy→light gas system is characterized (at least for small angles of incidence of the SSW) by the occurrence of a shock-wave configuration for which the SSW, the refracted shock wave, and the reflected rarefaction wave converge at the contact point in a coordinate system tied to this point. We can track the development of this configuration as the angle of incidence $\varphi_{1,\mathrm{H}}$ increases, using the following system of equations (the Rankine–Hugoniot condition) to calculate the parameters behind the SSW (j = H) and the refracted shock wave (j = L):

$$p_{1j} = \frac{2\rho_{0,j}\, q^2}{a_j + 1} \sin^2(\varphi_{1,j}) - \frac{a_j - 1}{a_j + 1} p_{0,j}\,, \quad a_j = \begin{cases} k\,, & j = \mathrm{H}\,, \\ m\,, & j = \mathrm{L}\,, \end{cases}$$

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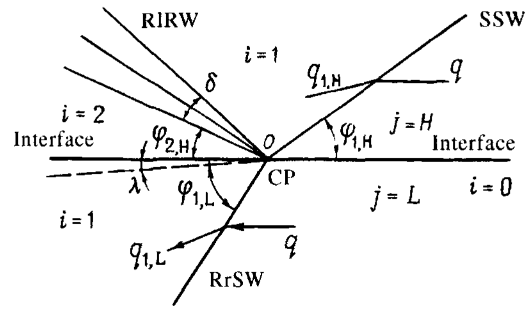

Fig. 1. Scheme of break decay in emergence of a skew shock wave at a heavy→light gas interface.

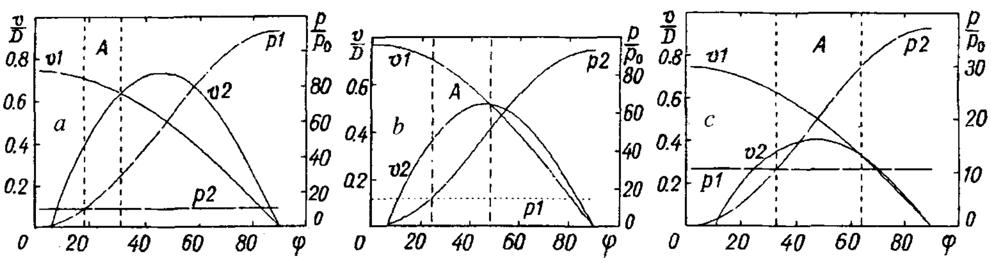

Fig. 2. Diagrams of the change in the pressure (p) and the component of the velocity normal to the interface (v) behind an SSW (1) and the refracted shock wave (2) in the light→heavy gas system (air–krypton) for $\varphi_{1,H}$ = 30° (a), 45° (b), and 60° (c). The relation of the parameters in region A permits the appearance of a reflected shock wave.

$$K\left(\frac{p_{1,j}}{p_{0,j}}\right)=\frac{a_j+1+(a_j-1)\frac{p_{1,j}}{p_{0,j}}}{a_j-1+(a_j+1)\frac{p_{1,j}}{p_{0,j}}},\quad \rho_{1,j}=\frac{\rho_{0,j}}{K\left(\frac{p_{1,j}}{p_{0,j}}\right)},$$

$$q_{1,j}=q\cos(\varphi_{1,j})\sqrt{\left(1+\tan^2(\varphi_{1,j})\,K^2\left(\frac{p_{1,j}}{p_{0,j}}\right)\right)}, \tag{1}$$

$$\vartheta_{1,j}=\varphi_{1,j}-\arccos\sqrt{\left(\frac{1}{1+\tan^2(\varphi_{1,j})\,K^2\left(\frac{p_{1,j}}{p_{0,j}}\right)}\right)},$$

where $q = D/\sin\varphi_{1,H}$, and k, and m are the polytrope indices of the light and heavy gases, respectively. The Prandtl–Mayer parameters of the flow behind the front of the reflected rarefaction wave can be determined by the corresponding system of equations [3, 4], which is specially transformed for this specific case:

$$q_{2,H}=c_{1,H}\sqrt{\left(2\frac{1+\alpha}{k-1}\left(\frac{1+\alpha}{\xi^{\frac{k-1}{k}}}-1\right)\right)},$$

$$\vartheta_{2,H}=\vartheta_{1,H}+\lambda_H\left(\arctan\frac{\sqrt{M_{2,H}^2-1}}{\lambda_H}-\arctan\frac{\sqrt{M_{1,H}^2-1}}{\lambda_H}\right)+$$

$$+\arctan\sqrt{M_{1,H}^2-1}-\arctan\sqrt{M_{2,H}^2-1},$$

$$\rho_{2,H}=\xi^{\frac{1}{k}}\rho_{1,H}\,,\quad p_{2,H}=\xi p_{1,H}\,,\quad \lambda_H=\sqrt{\left(\frac{k+1}{k-1}\right)}, \tag{2}$$

where

$$M_{1,H}=\frac{q_{1,H}}{c_{1,H}}\,,\quad M_{2,H}=\frac{q_{2,H}}{c_{2,H}}=\frac{2}{k-1}\left(\frac{1+\alpha}{\xi^{\frac{k-1}{k}}}-1\right); \tag{3}$$

$$\alpha=\frac{k-1}{2}M_{1,H}^2\,,\quad c_{1,H}=\sqrt{\left(k\frac{p_{1,H}}{\rho_{1,H}}\right)}.$$

The evolution of a complete system of equations that consists of (1) for j = H, L, (2), and (3) yields twelve equalities for determining fourteen unknowns: $p_{1,H}$, $p_{2,H}$, $p_{1,L}$, $\rho_{1,H}$, $\rho_{2,H}$, $\rho_{1,L}$, $q_{1,H}$, $q_{2,H}$, $q_{1,L}$, $\vartheta_{1,H}$, $\vartheta_{2,H}$, $\vartheta_{1,L}$, $\varphi_{1,L}$, and ξ (excluding all intermediate relations and notation). This system (in what follows, it will be characterized as complete) is closed by conditions that require equality of the pressure and the velocity component normal to the interface of the flows on the two sides of the interface:

$$p_{2,H}=p_{1,L}\,,\quad q_{1,L}\sin\vartheta_{1,L}=q_{2,H}\sin\vartheta_{2,H}\,. \tag{4}$$

The first of equalities (4) enables us to establish a relationship between $\varphi_{1,L}$ and ξ:

$$\varphi_{1,L}=\arcsin\sqrt{\left(\frac{(m-1)\,p_0}{2\rho_{0,L}\,q^2}\left(\xi\frac{p_{1,H}}{p_0}\lambda_L^2+1\right)\right)}, \tag{5}$$

where

$$\lambda_L=\sqrt{\left(\frac{m+1}{m-1}\right)}. \tag{6}$$

The second equation of (4) yields a transcendental relation for calculating ξ.

A numerical analysis of the complete system enables us to establish that the shock-wave configuration that occurs in this case depends, just as in [1], on two characteristic angles – φ_c and φ_t. The first of them (φ_c) fixes the moment at which the supersonic regime of flow behind the SSW changes to a subsonic one and is calculated for polytropic gases from the relation [1]

$$\operatorname{ctan}\varphi_c\geq\sqrt{\left(\frac{2kK\left(\frac{p_{1,H}}{p_0}\right)}{k+1}-\frac{k-1}{k+1}\frac{kp_0}{\rho_{0,H}D^2}K\left(\frac{p_{1,H}}{p_0}\right)-K^2\left(\frac{p_{1,H}}{p_0}\right)\right)}, \tag{7}$$

and φ_t is the angle of total refraction [1], for which only two plane waves (the SSW and the refracted shock wave) are in contact at the contact point. It can be calculated numerically from (1) for each specific case for j = H, L with the following conditions on the two sides of the interface:

$$p_{1,H}=p_{1,L}\,,\quad q_{1,L}\sin\vartheta_{1,L}=q_{1,H}\sin\vartheta_{1,H} \tag{8}$$

after replacement of $\varphi_{1,H}$ by φ_t. The calculation of φ_t was considered earlier [1, 2] in detail, and therefore without going into the details of the computations we only note that for the heavy→light gas system, unlike the system considered in [1, 2], the relation $\varphi_t < \varphi_c$ always holds (for the system krypton–air, $\varphi_t \approx 35^o$, air–methane, $\varphi \approx 38^o$, and CO_2–neon, $\varphi_t \approx 29^o$ while the corresponding φ_c are determined by values of about 60^o [5]).

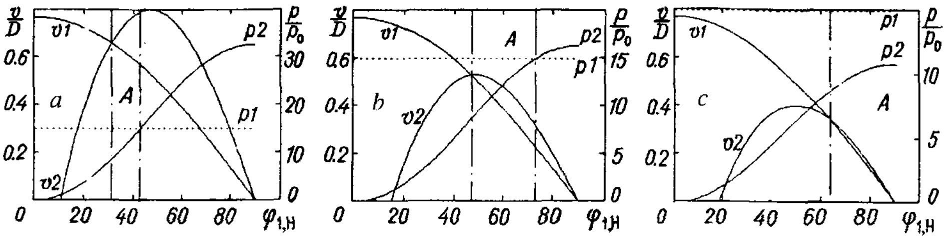

Fig. 3. Diagrams of the change in the pressure (p) and the component of the velocity normal to the interface (v) behind an SSW (1) and the refracted shock wave (2) in the heavy→light gas system (air–methane) for $\varphi_{1,H} = 30^o$ (a), 45^o (b), and 60^o (c). The relation of the parameters in region A permits the appearance of a reflected rarefaction wave.

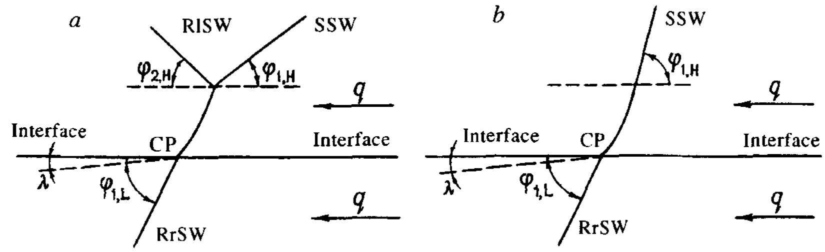

Fig. 4. Scheme of weak irregular regimes: a) shock regime; b) shockless regime.

Actual solutions of the complete system are possible only in the region of small angles ($\varphi_{1,H} < \varphi_t$). As soon as $\varphi_{1,H} = \varphi_t$ the bundle of Mayer–Prandtl characteristics disappears, giving way to a two-wave structure (the SSW and a refracted shock wave). Next, when $\varphi_{1,H} > \varphi_t$ a reflected rarefaction wave is impossible now and the conditions for the occurrence of a reflected shock wave are also absent (Fig. 3), and equilibrium in the vicinity of the contact point can be attained if it is assumed, just as in [1, 2, 5], that the contact point separates from the interface, forming a triple point above it with a reflected shock wave. Since $\varphi_{1,H} > \varphi_t$ only a weak [1] regime of interaction is realized (Fig. 4a). As $\varphi_{1,H}$ increases further, the configuration changes in complete agreement with [1, 2, 5]; as soon as $\varphi_{1,H} \geq \varphi_c$ the reflected shock wave disappears and a weak shockless irregular regime [1] is realized (Fig. 4b) which is characteristic of the heavy→light gas system in the entire range of $\varphi_c \leq \varphi_{1,H} \leq \pi/2$. This regime does exist for $\varphi_{1,H} = \pi/2$ (a glancing shock wave). A calculation of these shock-wave configurations is described in detail in [1, 5] and undergoes no alterations; for a glancing shock wave, the calculations of [6] hold.

All the aforesaid permits the conclusion that in interaction of shock waves with a heavy→ light gas interface, just as for the case of a light→heavy gas interface, for angles of incidence that exceed the angle of total refraction φ_t an irregular regime of interaction accompanied by formation of a curvilinear Mach wave is very likely to occur.

NOTATION

D, shock-wave velocity; v, component of the gas-flow velocity normal to the interface in a coordinate system tied to the contact point; c, velocity of sound in the medium; p, pressure; q, total velocity of the flow in a coordinate system tied to the contact point; k, polytrope index of the heavy gas; m, polytrope index of the light gas; M, Mach number; K, a, λ_H, λ_L, and α, functions defined in the corresponding references; ρ, density; φ, angle made by the wave with the interface; λ, angle of rotation of the interface behind the contact point (always without a subscript); ϑ, angle of rotation of the vector of the total velocity of the flow behind the shock-wave front. Subscripts: H, upper part of the plane from the interface level and higher; L, lower part of the plane from the interface level and lower; *j*, takes on the values of H and L; c, critical angle of change in the regime of flow behind the SSW from supersonic to subsonic; t, angle of total refraction.

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